

Large scale optimization: which Sigma works?

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Context

- R in Finance 2018:
 - Minimum Regularized Covariance Determinant Estimator – Kris Boudt
- Can we integrate this method in our optimization framework?
- Practitioners need to find methods that:
 - return consistent results
 - are easy to integrate
 - can run in a reasonable time

Portfolio optimization & obstacles

- Focus on Global Minimum Variance
 - Find the vector of weights that minimizes $w^T \Sigma w$
 - Only input to the optimization program is Σ , it needs to be:
 - invertible
 - a good estimator of future volatility
- In practice
 - Examples often involve small universes
 - The shorter your lookback, the longer your back-test

Which Sigma works?

- Covariance matrices formed from ‘fat data’ usually don’t solve
- Quadratic Solver (quadProg) requires Σ to be inverted
- For each universe, looking back at 1 year of trading days

		S&P100	S&P500	R2000
<i>base</i>	Sample	✓		
<i>base</i>	EWMA	✓		
<i>ledoit/wolf</i>	Shrinkage	✓	✓	✓
<i>rrcov</i>	MRCD	✓	✓	✓
<i>rmgarch</i>	DCC-GARCH	✓		

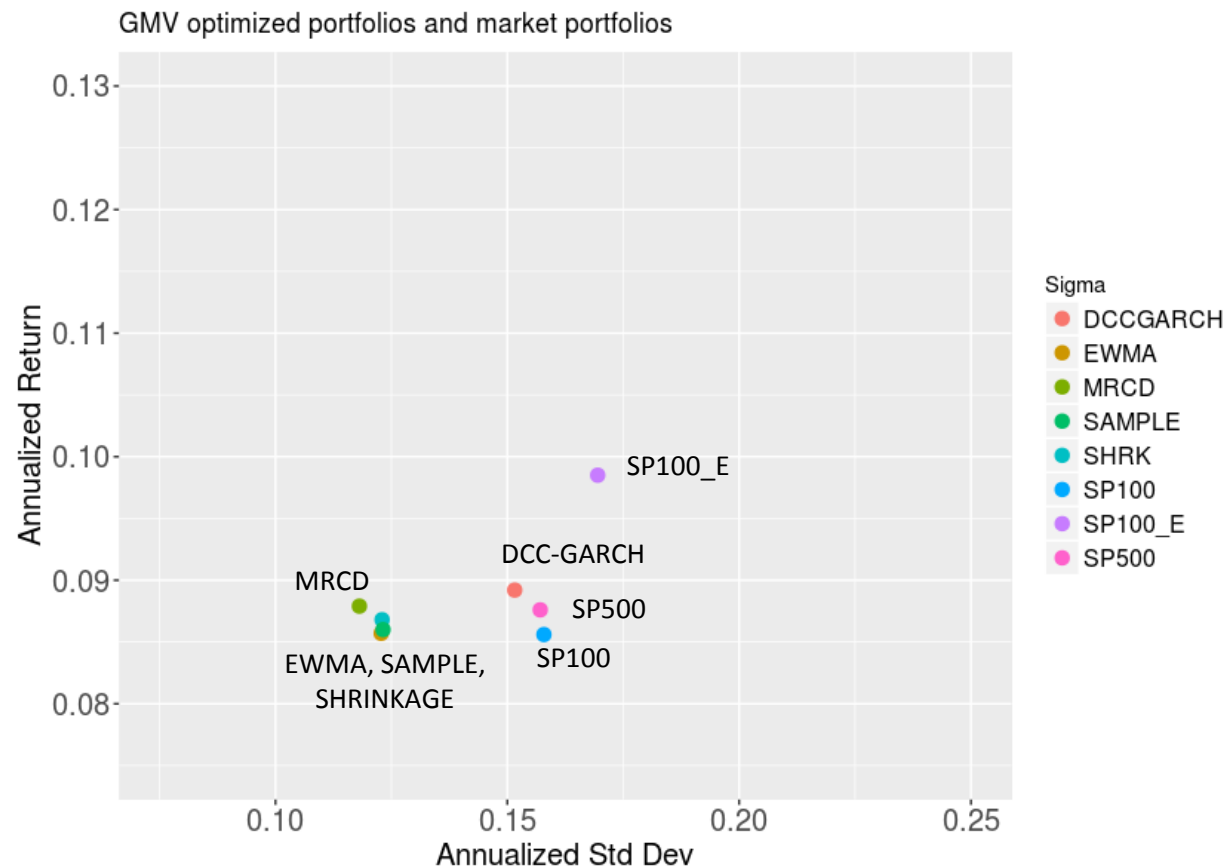
Computing performances

- How easy will it be to test as we implement?
- RAM = 240GB, CPU = Intel(R) Xeon(R) @ 2.50GHz x 12 cores

	S&P100	S&P500	R2000
Sample	0.01 sec	x	x
EWMA	0.01 sec	x	x
Shrinkage	0.03 sec	0.19 sec	1.51 sec
MRCD	2.85 sec	1.09 min	15.21 min
DCC-GARCH	3.12 min	x	x

Which Sigma works well – large cap?

- S&P100 from January 1996 to March 2019, rebalanced monthly
- One month forward estimate, under fully invested constraint $\sum_i w_i = 1$

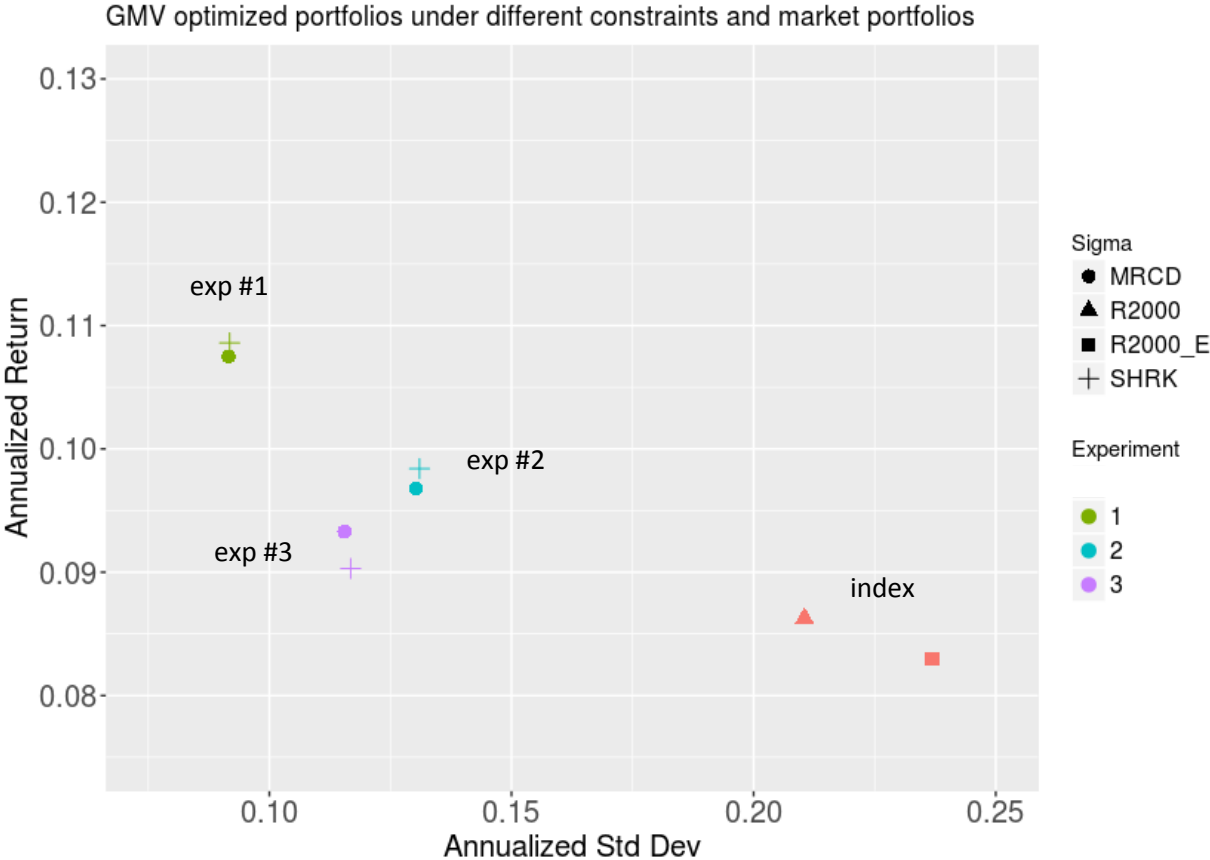


Portfolio	Sharpe	Std Dev
MRCD	0.74	0.118
SHRK	0.71	0.123
SAMPLE	0.70	0.123
EWMA	0.70	0.123
DCCGARCH	0.59	0.152

Index	Sharpe	Std Dev
S&P100 EQW	0.58	0.169
S&P500	0.56	0.157
S&P100	0.54	0.158

Which Sigma works well – small cap?

- R2000 (same back-testing characteristics)
 - Experiment #1: full investment
 - Experiment #2: full investment, long only
 - Experiment #3: full investment, long only, maximum 1% per position



Portfolio	Sharpe	Std Dev	# trades
SHRK_EXP1	1.18	0.09	9,363
MRCD_EXP1	1.17	0.09	9,000
SHRK_EXP2	0.77	0.12	477
MRCD_EXP2	0.81	0.12	611
SHRK_EXP3	0.75	0.13	684
MRCD_EXP3	0.74	0.13	777

Index	Sharpe	Std Dev	# trades
R2000 EQW	0.35	0.24	0
R2000	0.41	0.41	0

Conclusion

- When days > securities, all Sigma estimators work
 - Risk is reduced from the market portfolios
 - Straightforward to implement, with the exception of DCC-GARCH
- When securities >> days, only MRCD and Shrinkage work
 - Performance can become an obstacle
 - Constraints do not affect the two methods in relation to one another
 - Turnover and transaction costs need to be taken into consideration
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