Inference on the asset with maximal Sharpe ratio

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May 16, 2019

A common problem

A cariacature of quant work:

1. Backtest 1000 strategies.
2. Trade the strategy with maximal Sharpe ratio.
3. When your fund fails, go back to step 1 with a new fund, but backtest $10\times$ as many strategies.

So, formally:

- Suppose you observe $\hat{\zeta}$, a p-vector of the Sharpe ratios of some assets.
- Reorder assets so the first has highest Sharpe ratio: $\hat{\zeta}_1 = \max_i \hat{\zeta}_i$.
- Perform inference on the signal-noise ratio of that asset, ζ_1.
- Consider general case where the asset returns have correlation R.

You could just ...

Use a simple correction for Multiple Hypothesis Testing (MHT):[2]

- Assume the $\hat{\zeta}_i$ are independent, and all ζ_i are equal (to zero, say).
- Use MHT correction, like Bonferroni, when hypothesis testing.
- Or use beta correction for inference:

```
# cdf & qtile, max of np independent SRs, SNR=zeta
pmaxsr <- function(q, df, np, zeta = 0, ope = 1) {
    pv <- SharpeR::psr(q, df = df, zeta = zeta, ope = ope)
    pbeta(pv, shape1 = np, shape2 = 1)
}
qmaxsr <- function(p, df, np, zeta = 0, ope = 1) {
    pv <- qbeta(p, shape1 = np, shape2 = 1)
    SharpeR::qsr(pv, df = df, zeta = zeta, ope = ope)
}
```

But: Assume $\hat{\zeta}_i$ are independent, and ζ_i are equal. Low power!

California Doctor Says Throw This Test Out Now!

Conditional Estimation

Observe: when R is correlation of returns, then

$$\hat{\zeta} \approx \mathcal{N}\left(\zeta, \frac{1}{n}\left(\mathsf{R} + \frac{1}{2}\operatorname{Diag}\left(\zeta\right)\left(\mathsf{R} \odot \mathsf{R}\right)\operatorname{Diag}\left(\zeta\right)\right)\right). \tag{1}$$

- Max condition $\hat{\zeta}_1 \geq \hat{\zeta}_2, \hat{\zeta}_1 \geq \hat{\zeta}_3, \ldots$ can be expressed as $\mathsf{A}\hat{\boldsymbol{\zeta}} \leq \boldsymbol{b}$.
- Inference on ζ_1 of the form $\boldsymbol{\eta}^\top \boldsymbol{\zeta}$, with $\boldsymbol{\eta} = \boldsymbol{e}_1$.

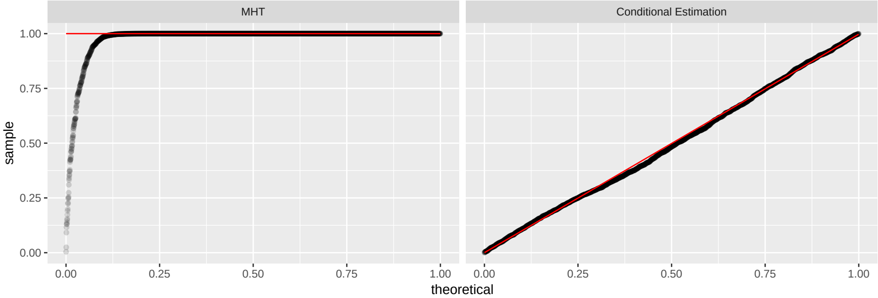
Use results of Lee *et al.* [1], to perform conditional inference on $\boldsymbol{\eta}^\top \boldsymbol{y}$, with $\boldsymbol{y} \sim \mathcal{N}\left(\boldsymbol{\mu}, \Sigma\right)$ conditional on $\mathsf{A}\boldsymbol{y} \leq \boldsymbol{b}$.
Main idea: "invert" $\Sigma^{1/2}$ transform to get a normal z linear in $\boldsymbol{y}$, such that the conditional part restricts z to a line segment, $\left[\mathcal{V}^-, \mathcal{V}^+\right]$.
Then get a p-value via

$$p = \frac{\Phi\left(z\right) - \Phi\left(\mathcal{V}^-\right)}{\Phi\left(\mathcal{V}^+\right) - \Phi\left(\mathcal{V}^-\right)}.$$

Also use this to compute confidence intervals.

Simulations

```
R <- pmin(diag(100) + 0.7, 1)  # mutual correlation of 0.7
mu <- seq(-0.1, 0.1, length.out = nrow(R))
set.seed(1234)
simvals <- replicate(2500, {
    X <- mvtnorm::rmvnorm(n = 5 * 252, mean = mu, sigma = R)
    max(colMeans(X)/apply(X, 2, sd))
})
```



Pros and Cons of this Test

Pro:

- Inference only operates on ζ_1, ignores $\boldsymbol{\zeta}_{2:p}$.
- Takes into account correlation of returns.
  Together these give higher power.
- The R need not be PD, so in theory this works for $n \ll p$.

Con:

- Have to estimate the covariance of $\hat{\boldsymbol{\zeta}}$.
- The normal approximation $\hat{\boldsymbol{\zeta}} \approx \mathcal{N}\left(\cdots\right)$ may not be good enough.

What's next?

To learn more about this work, or testing the signal-noise ratio in general:

- I hope to push this functionality to SharpeR.
- Check out my blog, http://blog.sharperat.io.
- Read my "Short Sharpe Course" on SSRN.[3]

Thank you.

Bibliography I

[1] Jason D. Lee, Dennis L. Sun, Yuekai Sun, and Jonathan E. Taylor. Exact post-selection inference, with application to the lasso, 2013. URL http://arxiv.org/abs/1311.6238. cite arxiv:1311.6238Comment: Published at http://dx.doi.org/10.1214/15-AOS1371 in the Annals of Statistics (http://www.imstat.org/aos/) by the Institute of Mathematical Statistics (http://www.imstat.org).

[2] Marcos López de Prado and David H. Bailey. The false strategy theorem: A financial application of experimental mathematics. *American Mathematical Monthly, forthcoming*, 2018.

[3] Steven E. Pav. A short Sharpe course. Privately Published, 2017. URL https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3036276.